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Edge charges and the Hall effect in semiconductors

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Received 26 July 1993

Abstract. A non-equilibrium distribution of carrier concentration across the sample caused by the Lorenz force is the origin of the Hall electric field. This distribution as well as the electrical and chemical potential profiles in the Hall direction are calculated for both three- and two-dimensional electron gases. In the three-dimensional case the deviation of electron concentration from its equilibrium value is rather small and located only near the sample edges. In the two-dimensional case the difference in electron concentrations at opposite sample edges is much more considerable and does not saturate with the sample size. At high magnetic field when the quantum Hall effect occurs, the difference in Fermi quasi-levels at the sample edges must be equal to the Landau level separation times an integer in large intervals of applied voltage, which has been observed in the experiment.

1. Introduction

Most measurements of the Hall effect in semiconductors are performed in the open-circuit condition where the Hall electric field appears along the axis (say, y axis) normal to the electric current and the magnetic field. In real experiments the Hall electric potential $\phi(y)$ at opposite sample edges may differ by several volts and even more, which exceeds considerably the characteristic energy of carriers (thermal or Fermi energy). Of course, this does not mean that the carrier density $n(y)$ is distributed in this potential field according to the Fermi function and, therefore, is extremely inhomogeneous. The explanation lies in the fact that the carrier distribution is governed not by the electrostatic but by the chemical potential ζ . Most people considering the Hall effect theory simply suppose ζ and, hence, n to be coordinate-independent, which is definitely not the case. There must exist local deviations from charge neutrality since it is necessary to have some charges causing the Hall electric field. For the particular case of quantum Hall effect the principal possibility of non-zero $\nabla\zeta$ was mentioned in [1, 2] but the effect was claimed to be non-essential.

The aim of the present paper is to find the actual distribution of the carrier concentration, chemical and electrostatic potentials in a semiconductor while undertaking Hall measurements and to discuss the effects resulting from violation of local neutrality. We shall show that under some circumstances these effects can be noticeable and interesting. Both three- and two-dimensional semiconductors with voltage V applied along the x axis and magnetic field $B \parallel z$ will be considered.

Our theoretical analysis will be based on the equation

$$\sigma_{xx} \left(-\frac{\partial\phi}{\partial y} + \frac{1}{e} \frac{\partial\zeta}{\partial y} \right) + \sigma_{xy} \left(-\frac{\partial\phi}{\partial x} + \frac{1}{e} \frac{\partial\zeta}{\partial x} \right) = 0 \quad (1)$$

expressing the absence of Hall current (σ_{xx} and σ_{xy} are the diagonal and non-diagonal components of the conductivity). This equation, combined with the Poisson equation, allows

us to find the space distribution of electrostatic and chemical potentials as well as of the carrier density. We shall assume that the sample length L in the direction of the current is much more than the width b in the Hall direction, which is typical for the samples for Hall measurements. In this case the above-mentioned concentration inhomogeneity far from the current contacts will depend only on the y coordinate and we simplify equation (1) by assuming $\partial\zeta/\partial x = 0$ and $\partial\phi/\partial x = -E_x = V/L = \text{const.}(\mathbf{r})$.

2. Three-dimensional sample

Consider first a three-dimensional sample. We shall see that in this case the carrier density fluctuations $n(y) - n_0$ (n_0 is the equilibrium electron density equal to the non-compensated impurity concentration) are not large, which allows us to expand ζ in terms of $(n - n_0)$:

$$\frac{d\zeta}{dy} = \left(\frac{d\zeta}{dn} \right)_0 \frac{dn}{dy}$$

((...)₀ means the value calculated at $n = n_0$) and write equation (1) in the form

$$\frac{\partial\phi}{\partial y} = \frac{1}{e} \left(\frac{d\zeta}{dn} \right)_0 \frac{dn}{dy} + \frac{\mu B}{c} E_x. \quad (2)$$

We have taken into account that $\sigma_{xy} = (\mu B/c)\sigma_{xx}$ where μ is the carrier mobility. By substitution of equation (2) into the Poisson equation

$$d^2\phi/dy^2 = (4\pi e/\kappa)[n(y) - n_0] \quad (3)$$

where κ is the dielectric constant, we obtain the final equation for $n(y)$:

$$d^2n/dy^2 = l^{-2}[n(y) - n_0]. \quad (4)$$

Here $l = [\kappa(d\zeta/dn)_0/4\pi e^2]^{1/2}$ is the screening length. This equation must be solved with the boundary conditions $E_y(\pm b/2) = 0$ reflecting sample neutrality, which gives

$$n(y) = n_0 - \Delta n[\sinh(y/l)/\cosh(b/2l)] \quad (5)$$

where

$$\Delta n = (\kappa/4\pi el)(\mu B/c)E_x \quad (6)$$

is the characteristic difference of electron concentrations at opposite sample edges. It can be easily shown that this value of Δn corresponds to the following difference of the chemical potentials:

$$\Delta\zeta = \zeta(b/2) - \zeta(-b/2) = -(e\mu Bl/c)E_x \simeq eV_H l/b \quad (7)$$

where V_H is the Hall voltage.

The predicted difference of concentrations at opposite sample edges in three-dimensional samples has a relatively small amplitude (except for high-resistivity substances with small carrier concentration). We shall see, however, that for a two-dimensional electron gas (2DEG) the effect is much more noticeable.

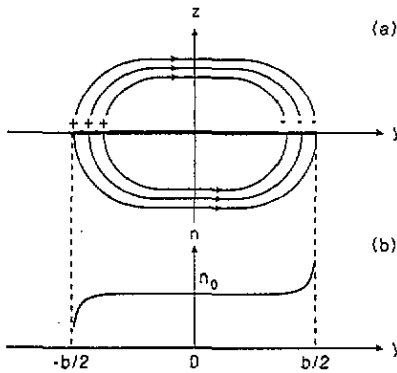


Figure 1. Schematic distribution of electric field (a) and carrier concentration (b) in the stripe of 2DEG during Hall measurements.

3. Two-dimensional sample

Consider first a structure with 2DEG in relatively low magnetic field when the Landau quantization can be ignored. The schematic character of the Hall electric field and charge distribution in the structure are shown in figure 1.

As before, the problem requires simultaneous solution of the continuity equation (1) (or (2)) and the Poisson (in this case Laplace) equation. However, for 2DEG the problem is much more complicated since the Hall potential distribution is now essential non-one-dimensional. By analogy with the theory of screening in 2DEG [3, 4], one must solve the Laplace equation $\partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2 = 0$ in the half-space $z > 0$ with the boundary conditions

$$(\partial\phi/\partial z)(y, 0) = (2\pi e/\kappa)[n(y) - n_0]. \quad (8)$$

(Note that here and further n represents the concentration per unit area and, hence, has different dimensionality than in section 2.) Assuming $\phi(0, 0) = 0$, we obtain by integrating equation (2) that

$$\phi(y, 0) - (\mu B/c)E_x y = (1/e)(d\zeta/dn)_0[n(y) - n_0] \quad (9)$$

which, after substituting into equation (8), gives eventually the boundary conditions for ϕ :

$$\begin{aligned} \frac{\partial\phi}{\partial z}(y, 0) &= l_2^{-1} \left(\phi(y, 0) - \frac{\mu B}{c} E_x y \right) & |y| \leq b/2 \\ \frac{\partial\phi}{\partial z}(y, 0) &= 0 & |y| > b/2. \end{aligned} \quad (10)$$

Here $l_2 = \kappa(d\zeta/dn)_0/(2\pi e^2)$ is the two-dimensional screening length. For a degenerate 2DEG l_2 is equal to half of the effective Bohr radius a_B , and expressions (9) and (10) are valid not only for small but for any Δn , which results from the constant density of states in 2DEG.

The Laplace equation with the boundary condition (10) resembles that of the theory of contact phenomena in 2DEG [4] and some conclusions can be made *a priori* by analogy. The screening capability of 2DEG is much weaker than in the bulk sample and any charge inhomogeneity will decrease very smoothly with the logarithmic divergence of the total screening charge. As a result, rather large effects can be observed even for samples with $b \gg l_2$.

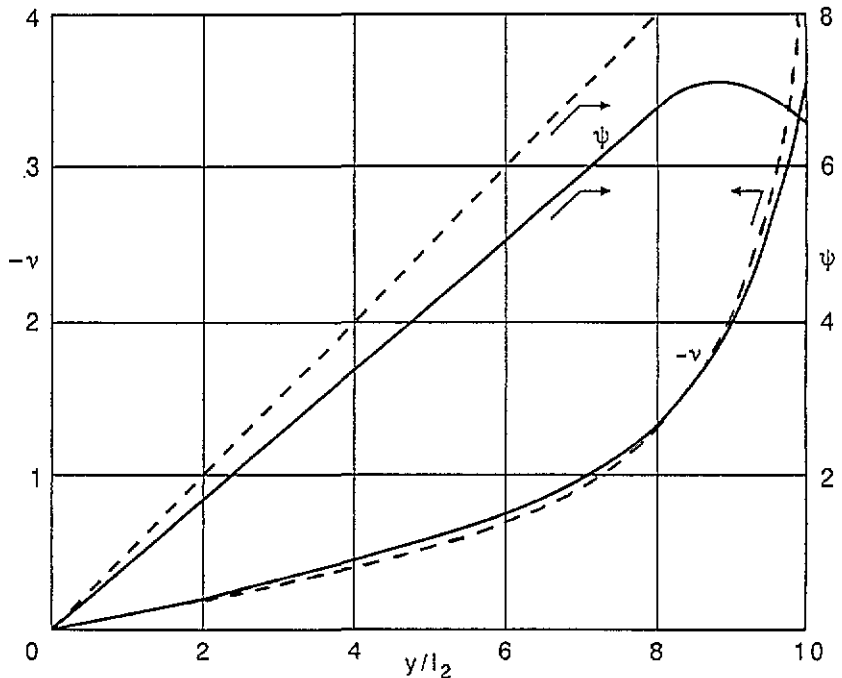


Figure 2. Coordinate dependence of the dimensionless charge density ν (left scale) and the dimensionless Hall potential ψ (right scale) in 2DEG with $\beta \equiv b/l_2 = 20$. The broken lines show these values in the absence of electrostatic effects ($l_2 \rightarrow 0$).

The equation cannot be solved analytically but can be transformed to an integral equation more convenient for numerical solution. We shall operate in terms of dimensionless units:

$$\xi = y/l_2 \quad \nu(\xi) = \frac{2\pi ec}{\kappa \mu B E_x} [n(y) - n_0] \quad \psi(\xi) = \frac{c\phi(y)}{l_2 \mu B E_x}.$$

In these units the solution of the Laplace equation with the boundary condition (8) is

$$\psi(\xi) = -\frac{2}{\pi} \int_0^\infty \frac{d\lambda}{\lambda} \sin(\lambda\xi) \exp(-\lambda z/l_2) \int_0^{\beta/2} \nu(\eta) \sin(\lambda\eta) d\eta \quad (11)$$

where $\beta = b/l_2$.

Assuming $z = 0$ and substituting equation (11) into equation (10) we obtain

$$\nu(\xi) = -\frac{2}{\pi} \int_0^\infty \frac{d\lambda}{\lambda} \sin(\lambda\xi) \int_0^{\beta/2} \nu(\eta) \sin(\lambda\eta) d\eta - \xi. \quad (12)$$

After integration over λ we obtain the final form of the integral equation:

$$\nu(\xi) = -\frac{1}{\pi} \int_0^{\beta/2} \nu(\eta) \log \left(\frac{\xi + \eta}{|\xi - \eta|} \right) d\eta - \xi. \quad (13)$$

This equation can be easily solved numerically by iteration, which gives us the concentration distribution along the Hall direction for 2DEG stripes of different widths (different β). After $\nu(\xi)$ is found, the potential distribution $\psi(\xi)$ and the Hall voltage $\psi(\beta/2) - \psi(-\beta/2)$ are determined by equation (11). Some of the final results are shown in figures 2 and 3.

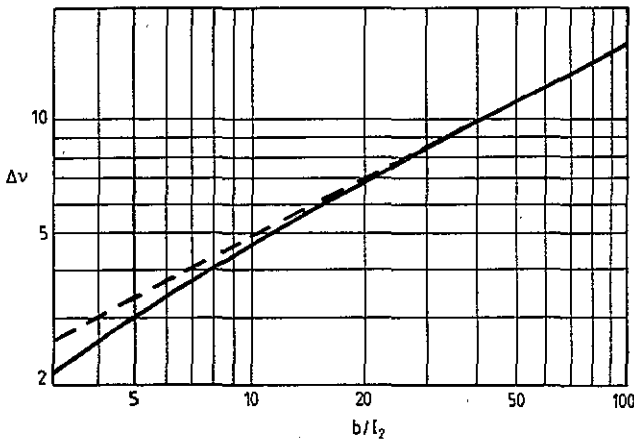


Figure 3. Dependence of the dimensionless concentration difference at opposite edges $\Delta\nu \equiv \nu(-\beta/2) - \nu(\beta/2)$ on the sample width. The broken line shows the dependence $\Delta\nu \sim b^{1/2}$.

In figure 2 the distributions of the electron concentration and the electrostatic potential are shown for a 2DEG structure with width considerably exceeding a_B , namely $b = 20l_2$. Noticeable deviations from neutrality are seen at fairly large distances from the sample edges whereas the Hall field even changes sign near the edges. Figure 3 demonstrates that, contrary to the three-dimensional case, the difference in electron concentrations at opposite edges does not saturate at large b , increasing approximately $\sim b^{1/2}$. To explain this fact, note that far from the sample edge we can neglect the diffusion current (the first term on the right-hand side of equation (2)). In this approximation the function $\psi(\xi)$ is linear whereas the charge density diverges near the edge:

$$\nu(\xi) = -[\xi/(\beta^2/4 - \xi^2)]^{1/2} \quad (14)$$

(see e.g. [5]). These dependences are shown in figure 2 by the broken lines. The diffusion term will provide an effective screening restricting the concentration increase at distance $\sim l_2$ from the edges. As a result, the charge density near the edge is of order $(b/l_2)^{1/2}(\kappa\mu B E_x/2\pi ec)$.

4. Quantum Hall effect

The effects described may play an important role also in the conditions of the quantum Hall effect. This case is characterized by the same geometry as shown in figure 1 and by the same boundary condition (8) to the Laplace equation, but the relation between n and ζ , and hence the final result, is rather different. Owing to the stepwise character of the n versus ζ dependence, the sample in the y direction will be divided into two types of regions:

(i) 'Metal' regions with ζ coinciding with some Landau level $\hbar\omega_c(N + 1/2)$ ($\omega_c = eB/mc$). In these regions $\nabla\zeta = 0$ and, according to equation (1),

$$-\frac{\partial\phi}{\partial y} = -\frac{\sigma_{xy}}{\sigma_{xx}} \frac{\partial\phi}{\partial x} \equiv E_H = \text{const.}(y)$$

which means that

$$\phi(y, 0) = -E_H y + C. \quad (15)$$

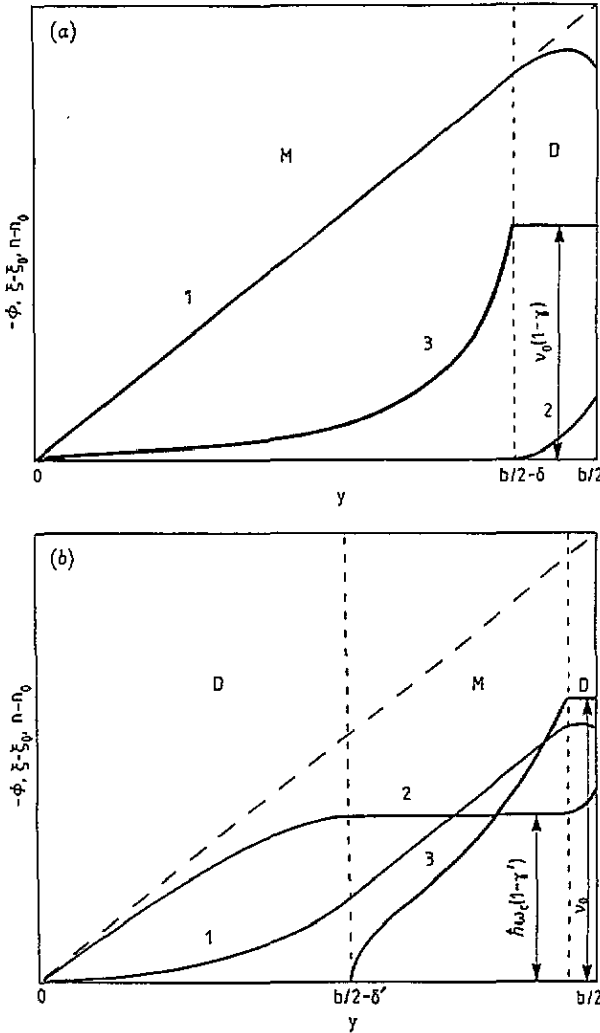


Figure 4. Schematic coordinate dependence of the electrostatic potential (curve 1), chemical potential (curve 2) and non-compensated electron density (curve 3) under quantum Hall conditions. (a) Non-integer equilibrium filling factor: $n_0 = \nu_0(N + \gamma)$, $\xi_0 = \hbar\omega_c(N + 1/2)$. (b) Integer filling factor: $n_0 = \nu_0(N + 1)$, $\xi_0 = \hbar\omega_c(N + 1/2 + \gamma')$ where $0 < \gamma, \gamma' < 1$. Letters M and D note 'metal' and 'dielectric' regions, respectively. Broken lines correspond to the linear dependence $\phi(y) = -E_H y$.

(ii) 'Dielectric' regions with ζ lying in a gap between Landau level and $\sigma_{xx} = 0$. These regions are characterized by constant electron concentration and hence

$$(\partial\phi/\partial z)(y, 0) = \text{const.} \tag{16}$$

A similar picture has already been discussed in the theory of contact phenomena [4, 6]. However, an important distinction must be pointed out. In the theory of contact phenomena, electrons are in equilibrium and $\nabla\zeta = e\nabla\phi$. In our case the functions $\zeta(y)$ and $e\phi(y)$ are not equal and must be found separately.

Two qualitatively different situations are to be considered depending on the equilibrium Fermi-level position ξ_0 .

Let us discuss first the situation of non-integer filling factor when $\xi_0 \simeq \hbar\omega_c(N + 1/2)$ (figure 4(a)). In this case the central part of the sample will be 'metal' with a homogeneous electric field (15). But this cannot be the case across the whole sample since it would require infinite charge density near the edges (equation (14)†). Therefore, for any applied voltage

† Formally this corresponds to the results of the previous section where, owing to infinite density of states at the Landau level, $I_2 = 0$.

there will be 'dielectric' stripes near the edges. To estimate the width of these stripes δ , we use equation (14): δ can be determined as the value of $(b/2 - y)$ where the surface charge density is equal to the concentration of electrons (or vacant places) at the Landau level. This concentration is of order $\nu_0 \equiv eB/2\pi\hbar c$ and hence

$$\delta/b \sim [(\kappa\hbar c/e^2)(E_H/B)]^2. \quad (17)$$

For any reasonable values of E_H and B corresponding to the quantum Hall effect, the right-hand side of equation (17) is much less than unity. The exact distributions of electron density, electrical and chemical potential in the sample are calculated in appendix 1 and shown schematically in figure 4(a). A similar non-monotonic $\phi(y)$ dependence has already been obtained by numerical calculations in [7, 8].

It is interesting to note that the total charge concentrated in half of the 'metal' region is of order $\kappa E_H b$ whereas the charge in the edge 'dielectric' stripe has much smaller value $\sim e\nu_0\delta \sim \kappa E_H b(\delta/b)^{1/2}$.

The arguments given above demonstrate that even at low current (small E_H) the electron concentrations at opposite Hall edges differ by the finite value: $n(b/2) - n(-b/2) = \nu_0 \equiv eB/2\pi\hbar c$.

Our conclusions are adequate for not very large E_H . If the current (and hence E_H) is large enough, the chemical potential ζ varying at the length δ may reach the next Landau level, causing one more 'metal' stripe in the sample. In this case $n(b/2) - n(-b/2)$ becomes equal to $2\nu_0$ rather than ν_0 . Let us estimate the corresponding critical value of E_H . It can be shown that $\zeta(b/2) - \zeta_0 \sim eE_H\delta$. An additional stripe appears when this difference exceeds $\hbar\omega_c$, which, together with equation (17), gives us

$$(E_H)_{cr} \sim B(e^4/\hbar c^3 \kappa^2 mb)^{1/3}. \quad (18)$$

Further increase in E_H will cause the appearance of additional 'metal' stripes followed by new jumps in $n(b/2) - n(-b/2)$.

If ζ_0 lies in a gap between Landau levels, $\zeta_0 = \hbar\omega_c(N + 1/2 + \gamma')$ with $0 < \gamma' < 1$, the situation is rather different (figure 4(b)). At low currents when eV_H is less than $\hbar\omega_c$, there are no delocalized electrons at the Fermi level. In this case formally $l_2 = \infty$ and screening is provided only by the deformation of Landau wavefunctions [5], which is beyond our approach based on the local connection between electron density and the potential.

At higher currents the chemical potential level near one or both edges begins to be pinned to a Landau level and the local filling factor acquires a non-integer value (qualitatively this fact has already been pointed out [9]). These 'metal' regions provide effective screening and change the potential distribution drastically compared to the low-current case. Note that, as in the previous case, there must also exist narrow 'dielectric' regions near the edges (see appendix 2 and figure 4(b)).

Calculations of appendix 2 show that $\phi(y)$ in the neutral central part of the sample is given by the formula

$$\phi(y) = -\frac{E_H(b/2 - \delta')}{\pi} \int_0^{2\delta'/b} \left(\frac{t}{2\delta'/b - t} \right)^{1/2} \log \left(\frac{t+2}{t} \right) dt \quad (19)$$

where δ' is the width of edge 'metal' stripes assumed to be much less than $b/2$. In particular, at the boundary between 'dielectric' and 'metal' regions

$$\phi(b/2 - \delta') \simeq -E_H\delta'[0.2 + 0.5 \log(b/\delta')]. \quad (20)$$

To determine the unknown value δ' , we note that, at the point $y = b/2 - \delta'$, ζ must coincide with a Landau level and hence $\zeta - \zeta_0$ must be equal to $\hbar\omega_c(1 - \gamma')$ (or to $-\hbar\omega_c\gamma'$ at the opposite edge). Taking account of equation (A5), this gives us the equation

$$eE_H\delta'[2.4 + \log(b/\delta')] = eE_Hb - \hbar\omega_c \quad (21)$$

(we have assumed $\gamma' = 0.5$). Equation (21) has solutions compatible with our assumption $\delta' \ll b$ in a rather narrow interval of currents (e.g. $\delta' < 0.1b$ only for $\hbar\omega_c < eE_Hb < 2\hbar\omega_c$). This means that soon after eE_Hb exceeds $\hbar\omega_c$, the 'metal' region covers a considerable part of a sample. As a result, we come to the situation similar to that of non-integral filling factor. The only difference is that $n(b/2) - n(-b/2)$ is equal to $2\nu_0$ rather than ν_0 .

So, in the conditions of quantum Hall effect there is a large interval of Hall fields, $\hbar\omega_c/eB < E_H < (E_H)_{cr}$, where the electron concentrations at opposite sample edges differ by a constant value ν_0 times an integer.

5. MOS structures

The results presented can be directly applied to such types of two-dimensional structures as heterostructures and thin films. In MOS structures the problem is more complicated due to additional screening caused by the metal gate. Formally this means that the Laplace equation must be provided by an additional boundary condition: $\phi = \text{const}$ at the plane $z = d$, where d is the oxide thickness.

The presence of the metal gate will result in additional decrease of $\Delta\phi = \phi(b/2) - \phi(-b/2)$. For a hypothetical MOS structure with extremely thin dielectric $d \ll a_B$, $\Delta\phi$ will be $\sim a_B/d$ times less than without the gate. At the same time $\Delta\zeta$ will increase considerably and we may expect the difference Δn to be higher than predicted in the previous sections.

So, MOS structures and heterostructures with the same 2DEG conductivity at the same current will have different Hall electric fields and different induced electron density gradients. As a result, the breakdown of the quantum Hall effect, regardless of its exact physical nature, may differ in these structures.

6. Summary

In summary, we have shown that electrostatic effects may disturb the commonly used picture of the Hall effect, causing inhomogeneity in the space distributions of Hall electric field and carrier concentration, especially in structures with 2DEG.

These effects decrease the electrostatic potential difference between the sample edges $\phi(b/2) - \phi(-b/2)$. At first glance, this must decrease the measured value of Hall voltage and hence of the Hall constant R_H , causing the deviation of the latter from the classical value $(n_0ec)^{-1}$ even in the absence of quantum Hall effect. It is not the case, however, since the measured Hall voltage is determined by difference in electrochemical potentials $\phi - \zeta/e$ rather than in electrostatic ones (see e.g. [1]), and the decrease in $\Delta\phi$ is exactly compensated by the appearance of $\Delta\zeta$ caused by Δn .

The effect can be discovered by direct measurements of the concentration difference at opposite sample edges Δn . In the classical case Δn is proportional to the current through the sample j , whereas in the quantum case $\Delta n \simeq \text{const}(j) = \nu_0 N(N = 1, 2, \dots)$ for a wide interval of j . This value is of order of 10^{11} cm^{-2} for typical B used in the measurement of

the quantum Hall effect. This is a fairly large quantity that can be discovered in experiments allowing local determination of the electron concentration in different parts of the sample.

One of these experiments has already been performed [10]. The conductivity changes induced by a focused flux of non-equilibrium phonons in 2DEG in high magnetic field were investigated. This effect has an oscillating B dependence similar to that of the Shubnikov-de Haas effect, but the phase and even frequency of oscillations determined by the electron concentrations (chemical potentials) differ at opposite edges of the sample. Further analysis showed [11] that the phenomenon was connected not with inherent sample inhomogeneity but with the Hall effect that resulted from the changing sign of Δn with reversing current or magnetic field. The difference in local values of ζ at opposite sample edges appeared to be just of order $\hbar\omega_c$, in accordance with the present theory.

Acknowledgments

I am grateful to LJ Challis for active support, valuable comments and hospitality during my visit to the University of Nottingham. I am also indebted to M I Dyakonov for stimulating discussion.

Appendix 1.

Consider for simplicity the case of half-filled Landau level. In this case the surface charge density in the edge stripes is $\pm e\nu_0/2$ and the potential distribution $\phi(y)$ is an odd function. It is to be determined from the Laplace equation with the boundary conditions

$$\begin{aligned} \phi(y, 0) &= -E_H y & 0 < y < (b/2 - \delta) \\ \partial\phi(y, 0)/\partial\phi &= -\pi\nu_0/\kappa & (b/2 - \delta) < y < b/2 \\ \partial\phi(y, 0)/\partial z &= 0 & y > b/2. \end{aligned} \quad (\text{A1})$$

This equation can be transformed into the integral equation for charge density $\sigma(y)$ in the 'metal' region $|y| < (b/2 - \delta)$:

$$E_H = \frac{e\nu_0}{\kappa} \log\left(\frac{b^2/4 - y^2}{(b/2 - \delta)^2 - y^2}\right) - \frac{2}{\kappa} \int_{\delta-b/2}^{b/2-\delta} \frac{\sigma(y') dy'}{y' - y} \quad (\text{A2})$$

where the integral should be understood as the principal value.

Equation (A2) has the following solution (see e.g. [12]):

$$\begin{aligned} \sigma(y) &= \frac{-1}{2\pi[(b/2 - \delta)^2 - y^2]^{1/2}} \left[\kappa E_H (b/2 - \delta) + \frac{e\nu_0}{\pi} \int_{\delta-b/2}^{b/2-\delta} \right. \\ &\quad \left. \times \log\left(1 + \frac{b\delta - \delta^2}{(b/2 - \delta)^2 - t^2}\right) [(b/2 - \delta)^2 - t^2]^{1/2} \frac{dt}{(t - y)} \right]. \end{aligned} \quad (\text{A3})$$

The unknown parameter δ can be found from the condition that (A3) has no singularity at $y = b/2 - \delta$. For $\delta \ll b$ this gives

$$\delta = b(\kappa E_H/4e\nu_0)^2 \quad (\text{A4})$$

in accordance with the qualitative estimate (17).

Knowledge of the charge distribution in the whole sample allows calculation of the electric potential $\phi(y)$ by direct integration and then the chemical potential $\zeta(y)$ may be found from the relation

$$e\phi(y) - \zeta(y) + \zeta_0 = -eE_H y. \quad (\text{A5})$$

Appendix 2.

If ζ lies midway between Landau levels, the potential distribution $\phi(y)$ is, as before, antisymmetric. It is caused by some charge distribution $\sigma(y)$ in the 'metal' stripe $b/2 - \delta' < y < b/2$ and the oppositely charged stripe $-b/2 < y < -b/2 + \delta'$. The electric field inside these stripes is homogeneous and equal to E_H , which gives us the integral equation for $\sigma(y)$:

$$E_H = -\frac{4y}{\kappa} \int_{b/2-\delta'}^{b/2} \frac{\sigma(y')}{y'^2 - y^2} dy'. \quad (\text{A6})$$

For $\delta' \ll b/2$ we may replace $y'^2 - y^2$ by $b(y' - y)$ and solve the equation in a similar way as (A2):

$$\sigma(y) = -\frac{\kappa E_H}{2\pi} \frac{y + C}{[(y - b/2 + \delta')(b/2 - y)]^{1/2}}. \quad (\text{A7})$$

The constant C is determined from the condition that equation (A7) has no singularity at the inner boundary of the stripe ($y = b/2 - \delta'$) which gives eventually

$$\sigma(y) = -\frac{\kappa E_H}{2\pi} \left(\frac{y - b/2 + \delta'}{b/2 - y} \right)^{1/2}. \quad (\text{A8})$$

The expression (A8) diverges at $y = b/2$. This singularity is suppressed by formation of narrow 'dielectric' stripes with $\sigma = \pm ev_0/2$ near the sample edges. The situation is similar to the case $\zeta_0 \simeq \hbar\omega_c(N + 1/2)$ and, as has already been shown, the width and total charge of these stripes are very small. So, everywhere except in the vicinity of the edges we may ignore the existence of these stripes and use equation (A8).

The calculated charge distribution creates the following potential in the neutral part of the sample ($|y| < b/2 - \delta'$):

$$\phi(y) = \frac{2}{\kappa} \int_{b/2-\delta'}^{b/2} \log \left(\frac{y' - y}{y' + y} \right) \sigma(y') dy'. \quad (\text{A9})$$

Substitution of equation (A8) into equation (A9) gives us the expression (19).

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